Incomplete information and unobservable action

- Rival's price is unobservable
  (recall Green & Porter)

- Incomplete information about demand

- *Symmetric* information: Both firms incompletely informed

- Learning over time
  - Collecting information today in order to have more knowledge about demand tomorrow

- Strategic aspects of learning
  - A firm may try to disturb the other firm's learning today in order to affect future decisions

Model:

Two firms. Two periods.

Product differentiation. Price competition each period.
- Prices strategic complements.

Firms do not observe each other's prices.

Firms do not know the market demand function.

$$q_i = a - p_i + bp_j$$

- Firm A wants firm B to set a high price in period 2.
- Firm B will only set a high price in period 2 if it believes demand is high.
- Firm B may think demand is high if it has high sales in period 1.
- Firm A may set a high price today in order for firm B to believe demand is high.
- But also firm B reasons the same way about firm A.
- And each firm also knows the other firm manipulates its learning.
- Both firms set high prices in period 1 in order to manipulate each other's learning.
- But each firm is able to see through the other firm's manipulation and learns the correct demand condition before period 2.
- *Signal-jamming*: manipulating others' learning
- In our case: signal-jamming increases period-1 prices.

Signal-jamming

$$\underbrace{s}_{\substack{\text{observed}\\\text{by the other}}} = \underbrace{\alpha}_{\substack{\text{controlled}\\\text{by the firm}}} + \underbrace{\varepsilon}_{\substack{\text{stochastic}\\\text{term}}}$$

Other applications:
Organizational economics, corporate governance
– moral hazard

A specific model:

Firms: *I* and *II*
No costs.

Demand: $D_i(p_i, p_j) = a - p_i + p_j, \quad i \neq j.$

No firm knows a, only its expected value: $a^e = \mathrm{E}a$

The one-period case: (Benchmark)

Each firm solves:

$$\max_{p_i} E\pi_i = E\{(a - p_i + p_j)p_i\} = (a^e - p_i + p_j)p_i$$

Best-response function: $p_i = \dfrac{a^e + p_j}{2}$

Equilibrium: $p_I = p_{II} = a^e$.

The two-period case:

Learning about a if other firm's price is observable:

$$a = D_i + p_i - p_j$$

But other firm's price is not observable

$$\underbrace{D_i + p_i}_{\substack{\text{observed}\\ \text{by firm } i}} = \underbrace{p_j}_{\substack{\text{controlled}\\ \text{by firm } j}} + \underbrace{a}_{\substack{\text{stochastic}\\ \text{term}}}$$

In a symmetric equilibrium, each firm sets the same price in equilibrium, α, so that: $D_i = a - \alpha + \alpha = a$

But which price?

If firm *II* sets the price α and believes firm *I* does the same, what price would firm *I* want to set?

Firm *II*'s estimate of a after period 1:

$$\tilde{a} = D_{II}^1 = a - \alpha + p_I^1 \rightarrow \tilde{a} = \tilde{a}\left(p_I^1\right)$$

In period 2, firm *II* believes it is playing a game of complete information where $a = \tilde{a}\left(p_I^1\right)$.

$$\rightarrow p_{II}^2 = \tilde{a}\left(p_I^1\right)$$

What are the incentives for firm I to set a price in period 1 that differs from α?

First, consider period 2: Firm I has been able to deduce the true a and solves:

$$\max_{p_I^2}\left[a - p_I^2 + \tilde{a}\left(p_I^1\right)\right]p_I^2$$

$$\rightarrow\; p_I^2 = \frac{a + \tilde{a}\left(p_I^1\right)}{2} = \frac{a + a - \alpha + p_I^1}{2} = a + \frac{p_I^1 - \alpha}{2}$$

Firm I's period-2 profit:

$$\pi_I^2 = \left(a + \frac{p_I^1 - \alpha}{2}\right)^2$$

Period 1:

What is the optimum price for firm I in period 1, given firm II's price α?
Discount factor: $\delta \in (0, 1]$

Firm I solves:

$$\max_{p_I^1} \underset{a}{E}\left[\left(a - p_I^1 + \alpha\right)p_I^1 + \delta\left(a + \frac{p_I^1 - \alpha}{2}\right)^2\right]$$

FOC: $a^e - 2p_I^1 + \alpha + \delta\left(a^e + \dfrac{p_I^1 - \alpha}{2}\right) = 0$

In a symmetric equilibrium: $p_I^1 = \alpha$.

$$a^e - 2\alpha + \alpha + \delta a^e = 0$$

$\Rightarrow$ First-period price: $\alpha = a^e(1 + \delta)$

- Manipulation of learning fails.
- The firms set higher prices in period 1 than if manipulation of each other's learning were not possible.
- Puppy-dog strategy: A high price today in order for the other firm to believe demand is high and therefore set a high price tomorrow.

Strategic interaction in one market –
incomplete information in another

A version of *predation*:
The stronger firm competes aggressively in order to reduce the weaker firm's financial resources.

Product market: Duopoly – complete information

Capital market: Competitive – incomplete information

Two periods.

The two firms differ in financial strength:
The "long purse" story.

In order to operate in the market in period 2, each firm has to incur an investment K.

Firm 1 has internal funds in excess of K.

Firm 2 has to borrow on the capital market: Its internal funds equal $E < K$.

Firm 2 borrows $D = K – E$, and has to pay back: $D(1 + r)$

Interest rate: r

Firm 2's gross profit in period 2 is stochastic: $\tilde{\pi} \in [\underline{\pi}, \overline{\pi}]$

Cumulative distribution function: $F(\tilde{\pi})$; $F'(\tilde{\pi}) = f(\tilde{\pi})$

Expected value: π^e

If $\pi < D(1 + r)$, then firm 2 goes bankrupt.

Bankruptcy:
The bank receives π and incurs bankruptcy costs B.

Competitive capital market – banks' profits 0.

Banks' cost of funds: r_0

The interest rate in equilibrium solves:

$$(1+r)D[1-F(D(1+r))]+\int_{\underline{\pi}}^{D(1+r)}[\tilde{\pi}-B]f(\tilde{\pi})d\tilde{\pi}=(1+r_0)D$$

The expected bankruptcy costs will have to be covered by the borrowers.

So firm 2's capital costs is

$$[(1 + r_0)E] + [(1 + r_0)D + BF(D(1 + r))] =$$

$$(1 + r_0)K + BF((K - E)(1 + r))$$

Firm 2's expected net profit in period 2:

$W = \pi^e - (1 + r_0)K - BF((K - E)(1 + r))$

The higher is firm 2's internal funds, the more likely is it that firm 2 will undertake the period-2 investment:

An increase in E

- lowers debt $K - E$
- lowers interest rate r

Thus: $\frac{dW}{dE} > 0$

Period 1:

- E is a function of firm 2's period-1 profits.
- Firm 1 can lower E by reducing prices in period 1.
- Predatory pricing.